

# Multivariable Calculus — Assignment 5

## Surfaces and Partial Derivatives

Due Nov. 4, 2003

Solve the problems below. Be sure to include the reasoning that leads to your conclusions, when appropriate. *Maple* is fair game on any question, provided that you include and/or describe your work!

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- (a) Find the equation of the plane containing the points  $(2,4,1)$ ,  $(0,-6,-2)$  and the origin. In general, if a plane contains the origin as one of its points, what can you say about the equation of the plane?  
(b) Find equations for the line of intersection of the plane you found in (a), and the plane  $2x - 3y + z = -5$ , as well as the angle of intersection between the planes.  
(c) Are the two planes at right angles to each other? In general: if you have the equations of two planes, what would be the fastest way to determine whether the planes are perpendicular?

- Describe the domain of the function  $z = \frac{xy}{2 + \sqrt{x^2 + y^2 - 4}}$ , and sketch the domain on the  $xy$ -plane (on paper). Then try to get as good a plot as you can of the surface using *Maple*.

- Joe the Benevolent Despot rules his Kingdom from a castle set atop the Mythical Mountain at the centre of the land. His domain is  $(x, y) \in [-10, 10]$  (all units in miles), and is broken into four provinces defined by curved ridges that carve up the country. The lay of the land is defined by the following function:

$$z = \frac{2}{1 + \sqrt{|y - \sin x|} + \sqrt{|x + \sin y|}}$$

- Use *Maple* to get a good picture of Joe's kingdom, clearly showing the ridges that break up the region into four provinces. (NOTE: Use the `view` subcommand to convince *Maple* to show the peak, and `grid=[50, 50]` or even higher resolution to improve the quality of the plot.)
- Most functions have smooth surfaces, without any ridges. What part of the definition of this function causes ridges to occur?
- Give equations for the province boundaries, and plot a map of the kingdom with these boundaries in *Maple* (in 2 dimensions).
- Use *Maple* to get a relief map of the kingdom. Sketch the province borders in your plot with a pencil. Notice there are several small somewhat circular contour lines that would indicate a peak or a valley. Are there really peaks or valleys there, or is *Maple* having trouble? Provide a mathematical argument to support your position.
- All four of the provinces claim they have the hardest time getting to the castle, since the slope is very steep. Each province's path to the castle is along either the  $x$ - or  $y$ -axis (near the castle, at least). Investigate the limit of the slopes of the roads as you approach the castle in each of the four directions. Which province has it roughest, if any?

- The total resistance  $R$  of two resistors linked in parallel is given by the equation  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ .

(a) Resistor 1 is engineered to have a resistance of 10 ohms, while Resistor 2 has a resistance of 15 ohms. Resistor 1 is built to provide a maximum error in resistance of 5%, while Resistor 2 is better built and has a maximum error of 2%. Use differentials to approximate the total error of the pair of resistors (in ohms), and calculate the percentage error.

(b) Repeat part (a), but for resistors with resistance of 25 ohms each (same percentage error for each). Do you get the same cumulated percentage error this time?

5. Find all the partial derivatives of the following function. Also find all the second derivatives. You should find that two of them are the same! We'll see why, in a couple of weeks.

$$f(x, y) = e^{xy^2}$$