Multivariable Calculus — Assignment 2
Polar Coordinates

Due Friday, Oct. 3, 2003

Solve the problems below. Be sure to include the reasoning that leads to your conclusions, when appropriate. *Maple* is fair game on any question, provided that you include and/or describe your work!

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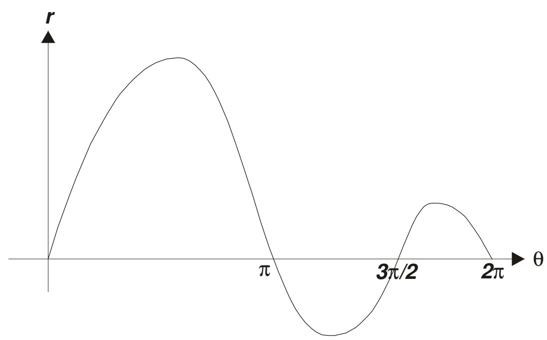
1. The curve $r = 1 + \cos\theta$ is called a cardioid because of its heart shape. If you place a constant *k* in front of the cosine term, a variety of shapes emerge.
(a) Let *k* vary from 0 upwards , and describe the different types of curve that emerge. What is the limiting curve as $k \to \infty$?
(b) For *k*>1, find the slope of the curve the first time it goes through the origin. What is the limiting value of this slope as $k \to \infty$?



2. A certain function $r = f(\theta)$ looks like the graph above when plotted on Cartesian $r-\theta$ axes. Supposing the function represents a curve in polar coordinates instead, sketch the curve.

3. Usually, the equation for a polar curve is defined as $r = f(\theta)$, but there is no particular reason why you might not come across the equation of a curve where *θ* is a function of *r*. You may wish to experiment with plotting some curves defined like this (see technology tips at the end of the assignment).
(a) Suppose *θ* is a linear function of *r*, say, $\theta = mr + b$, for $r \geq 0$. Plot some examples; what kind of curves do you get? What is the geometric meaning of *b*?
(b) Must every curve $\theta = f(r)$ pass through the origin? Explain why.
(c) Assuming that $r \geq 0$, show that any curve $\theta = f(r)$ can never intersect itself.
(d) For $\theta = 2r + \pi/2$, determine the smallest positive value of *r* for which the curve has a horizontal tangent. (HINT: remember how we found *dy/dx* in the normal polar case $r = f(\theta)$.)

4. Make up your own polar function using trigonometric functions and other basic mathematical tools. See if you can find something that looks pretty or interesting! When we're done, we'll have an "art gallery" of polar curves. Explore the nature of the function by changing it in a small way: for instance, place a coefficient in front of one of the terms (as in #1), or change slightly the argument of one of the trigonometric quantities. Explain what happens to your curve as you make

this change. (This project is given in an assignment at the U.S. Naval Academy each year. The most spectacular example one of the students there found was $r = 1 + \sin(t/24) + \cos 2t$. This is a neat example to plot!)

5.A favourite spot for hobbyists to sail their remote-controlled boats is at Snoopy Lake, which is defined by the polar equation $r = 400 + 100\sin 2\theta + 100\cos 3\theta$, where r is measured in metres.
(a) Make a plot of Snoopy Lake. Then, determine the values of θ for which the lake's shore is proceeding North-South or East-West (vertical/horizontal tangents). How far is it from the North point of the lake to the South point? (**Warning:** the direction of travel for this distance isn't due N/S.)
(b) Joe decides to get into remote-controlled boating. Just as he gets his boat started, he slips on a wet rock and drops his remote control into the lake! The boat has been put into a loopy path determined by the equation $r = 100\cos 2\theta$. Joe begins chasing his boat along the lakeshore counterclockwise, so that the value of θ is always the same for both him and the boat. Make a plot which contains both Snoopy Lake and the boat's path, and indicate by arrows the motion of the boat on the path. At what point is Joe closest to his boat? What is this minimum distance? Indicate the positions of Joe and the boat at this moment on your plot.

PLOTTING TIPS:

POLAR PLOTS IN MAPLE:

plot([function, theta, theta=….] , coords = polar);

MULTIPLE POLAR PLOTS IN MAPLE:

plot({ [first function, theta, theta=….] , [second function, theta, theta=…] }, coords=polar);

POLAR PLOTS IN MAPLE WHERE θ IS A FUNCTION OF r:

plot([r, your function, r = …], coords = polar);