

Mathematical Statistics — Assignment 4

Expectation

Due Oct. 22, 2003

The “Glen” questions below are more extended and challenging than those in the textbook. Please start this assignment early!

Textbook: p. 137 # 4.10, 4.20
p. 149 # 4.28, 4.34, 4.37, 4.53

Question Glen.1:

The probability that a roulette wheel stops on "red" is $18/38$, and spins are independent. For each dollar bet on red, you win a dollar if the wheel stops on red, and lose your bet otherwise.

(a) One possible strategy to win at roulette might be as follows: bet \$1 on red. If you win, you quit. Otherwise, you bet \$2 on red for the next spin. Continuing in this fashion, you quit if you win and double your bet if you lose. Explain why you cannot lose under this scenario. Then, explain why you won't book the next flight to Las Vegas and try this (i.e., what's wrong with the strategy, in practice?).

(b) Suppose you start with \$127, and perform the strategy in (a). If you lose seven times in a row, you will be bankrupt. Find the probability distribution of your net gain, and calculate the expectation.

(c) Suppose that you repeatedly use the strategy in (a). If you win, you restore your capital to its initial value and pocket the profit. You resolve not to gamble with your profits. What is the probability that you accumulate a profit of \$127 before your capital is wiped out?

Question Glen.2:

Suppose that n people take a blood test, and each has probability p of being infected with a disease, independently of the others. To save time, blood samples from k people are pooled and analyzed together. If this analysis gives a negative result, then none of the k people is infected, and the one test suffices for all k people. If the pooled test gives a positive result, then at least one of the k people is infected, and they must be tested [again] separately. In this case, $k+1$ tests are necessary for the k people.

(a) Show that, on the average, the number of tests required for a group of k people is

$$k + 1 - k(1 - p)^k .$$

(b) What is the expected number of tests required for the n/k groups of k people?

(c) Show that, if p is small, the expected number of tests in (b) is approximately $n(kp + 1/k)$, and is minimized for $k \approx 1/\sqrt{p}$.

(d) Make a table of the optimal values of k for various values of p , and describe in practical terms the conclusion of your study.