

Mathematical Statistics — Assignment 10

Final Synthesis — Hypothesis Tests/Confidence Intervals

Due Dec. 17, 2003

Please adhere to the guidelines on this final synthesis assignment. This means: **no other books, no conferring**. Just you, your notes, the textbook, Maple, and Excel if necessary.

1. Recall that the exponential density function is *memoryless*. One way to go beyond memoryless to allow for fatigue (no occurrence in the past increases the likelihood of one in the future) or stability (no occurrence in the past decreases the likelihood of one in the future) is to generalize to gamma distributions and vary α . For exponential (memoryless), $\alpha = 1$.
 - (a) For the waiting times between microearthquakes, it has been found that distribution that best fits this data is a gamma distribution with $\alpha = 0.509$ and $\beta = 869.6$. What are the mean and variance of these waiting times? (Units are in hours.)
 - (b) Is this an example of fatigue, or of stability? Explain.¹ In general, what values will α take if the situation is one of fatigue?
 - (c) Using the language of hypothesis testing, determine whether an observed waiting time of 65 hours between microearthquakes would cause us to doubt our assumption about the distribution of waiting times.
 - (d) Get a confidence interval for waiting times between sets of 50 microearthquakes.
2. Use Stirling's formula to demonstrate that the t distribution approaches the standard normal distribution when ν (the number of degrees of freedom) $\rightarrow \infty$.
3.
 - (a) Five measurements of the tar content of a certain kind of cigarette yielded 14.5, 14.2, 14.4, 14.3, and 14.6 mg/cigarette. The cigarettes are advertised to contain an average of only 14.0 mg/cigarette. Assume that the underlying distribution is normal. Does this data constitute evidence against the manufacturer's claim?
 - (b) Exercise 13.36. You'll only need techniques from class.
4.
 - (a) Suppose you have a 95% confidence interval for a mean, and you wish to cut its width in half. By what factor should your sample size be increased?
 - (b) If you wish to shrink the 95% confidence interval to half its previous length *without gathering more data*, you will be forced to decrease your confidence level. What confidence level will produce a confidence interval half of its original length?
5. Take some collection of data from the rest of your studies (perhaps a scientific experiment, or a psychological study). If you can't think of one, then generate your own data set to answer some question (support for Bush on campus, number of ice cream cones taken each hour in the dining hall, ... anything).
 - (a) Perform a statistical test on your data to answer the question that was posed. Use the language of hypothesis testing that we saw in class, and make the appropriate conclusion.
 - (b) Generate a confidence interval for your data as well, and make the appropriate conclusion. If it doesn't make sense to use a confidence interval for this data set, then get another data set.

¹ HINT: Look at a graph, and consider whether a recent microearthquake might cause an decreased or increased likelihood of a repeat event in the near future.