

Spherical Trigonometry — Assignment 4

Final Synthesis

Due May 20, 2004

This is a “synthesis” assignment, which implies the following: you may use your notes and Todhunter (although not for question 2); you must work on your own (no help from friends); you can get clarifications of questions from me, but not real hints.

1. Recall that the SSA (side-side-angle) case of a spherical triangle was difficult to solve using the standard four-element formulas (i.e., spherical laws of cosines and sines, and variants).
 - (a) Invent your own SSA triangle, and solve it using the four-element formulas.
 - (b) Then use one of our new formulas (the Delambre and Napier analogies) to help solve SSA triangles in a much more elegant manner.

2. We proved the spherical law of cosines essentially algebraically, following Todhunter. But it is possible to prove it geometrically, using essentially the same diagram that Yousuf came up with for his proof of the spherical law of sines:
 - (a) In the above diagram, identify which line segments are equal to the sines and cosines of a and b . Then give trigonometric expression for the lengths of EG, GO, DE, and DF (in terms of a , b , c , A , B , and C).
 - (b) From your results in (a), prove the spherical law of cosines.
 - (c) Also prove that $\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$.
 - (d) From the results of (b) and (c), and the spherical law of sines, demonstrate algebraically that $\sin A \cot B = \sin c \cot b - \cos c \cos A$ (one of the formulas we derived in class).

3.
 - (a) Demonstrate any of Delambre’s analogies 2, 3, or 4, using a method analogous to what we saw in class to show Delambre’s first analogy.
 - (b) Then, as we saw in class, derive any one of Napier’s analogies 1, 2, or 3, using a method analogous to what we saw in class.

4. Bennington’s latitude and longitude are $42^{\circ}55'N$ and $73^{\circ}14'W$ respectively. The web site www.astro.com/atlas allows you to look up the latitude and longitude of your home town.

- (a) From these quantities, determine the distance in miles from Bennington to your home town.
- (b) This summer, you want to fly from Bennington to visit me in Frankfurt¹, then home, then back to Bennington. How many miles will you fly?

¹ In full, its name is Frankfurt am Main (meaning, on the Main River).