Spherical Trigonometry — Assignment 3

Due May 6, 2004

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1. In this problem we will follow Ptolemy's method of determining the ortive amplitude of the Sun[1], and hopefully solve it two other ways. In the diagram below, H is the rising Sun, AE is the equator, BE is the horizon, and N is the celestial North Pole. The Sun will travel to the left and slightly up, until it reaches its highest altitude at noon, due South on the left edge of the diagram, just below A.

   (a) Explain why the spherical angle ANH is equal to $t/2$, where t is the length of daylight on this given day.[2]
   (b) Derive a formula for the amplitude a using the Law of Sines.[3]
   (c) Derive a formula for a using either Menelaus or R4Q.
   (d) Derive yet another formula for a, using Todhunter's general formula.
   (e) At Rhodes, where Hipparchus lived for part of his life, the shortest day (at the winter solstice) is 9.5 hours long. Where will the Sun rise on that day?
   (f) Ptolemy does not take into account the effect of atmospheric refraction in the above. Refraction has the effect of making the Sun appear higher in the sky than it actually is. What effect will this have on the results we obtain by our formula(s) above?

2. Our goal in this problem is to determine our local terrestrial latitude, given the length of the longest day of the year. We'll use the same diagram as above. Note that our latitude φ is the altitude of the celestial North Pole N above the horizon Δ.
   (a) Use Menelaus, or any other of our tools, to derive a formula for φ as a function of the amplitude a and the half length of daylight $t/2$.[4]
   (b) This isn't good enough yet, because it requires knowledge of the amplitude. Use the

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[1] The ortive amplitude is the arc along the horizon from the east point to the place that the Sun rises, arc EH in our diagram.
[2] Don't forget that arcs on the equator and time units are interchangeable: 360 degrees = 24 hours.
[3] Ptolemy's formula is $\cos a = \cos\delta \cdot \sin t/2$. Your result may appear to be different.
[4] Ptolemy's formula is $\sin\varphi = \cot a \cdot \tan t/2$. Your result may appear to be different.

formula for the amplitude in Question 1 to derive a formula for φ as a function of the Sun's declination δ and $t/2$.
(c) This is *still* not good enough, because it requires knowledge of the declination. Consider the situation on the longest day of the year; in particular, what is the value of δ on that day? Use this to come up with a final formula for φ that depends only on the length of the longest day and $\varepsilon\left(=23.5^\circ\right)$.
(d) Confirm your formula by plugging in the length of the longest day at Rhodes (14.5 hrs), and calculate its latitude. You should get about 36°.

3. We suspect that Dana "Carmen Sandiego" Brzezinski has stolen the fabulous Möbius Diamond Bracelet. At the interview, she claims to know nothing. "I was just on one of my jaunts around the world!", she asserts weakly. "A dubious story", we reply sarcastically. "Tell us about this so-called jaunt of yours." Dana tells us: "My journey was three direct plane flights, from Appleton to Bumblyburg to Crazyville, and finally back to my home in Appleton. When I reached Bumblyburg, I turned right toward Crazyville, making an angle of 75°. After reaching Crazyville I turned right again, making an angle of 95°. My return journey was 85° long. Now unhand me, you ungenteel coppers."
"Not so fast, you nefarious culprit! Your alibi holds about as much water as a Klein bottle (or as much ice cream as one of Julia's leaky containers)."
How do we know that Dana is lying?