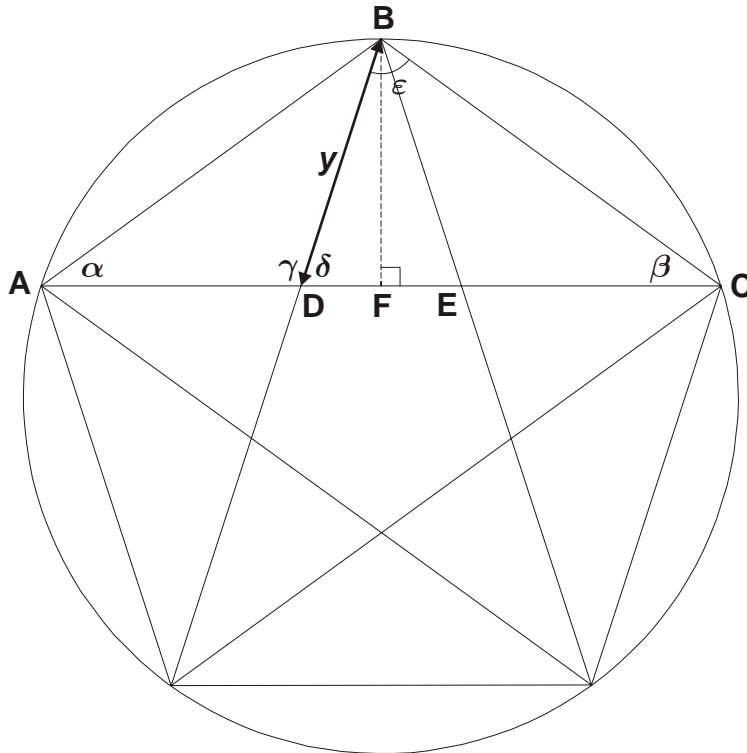


Spherical Trigonometry — Assignment 2

Due April 29, 2004

- To make up for my allowing you to use a calculator to determine $\sin 36^\circ$ last week, now I'd like you to derive it yourselves, according to the diagram below. In the diagram below I have drawn a regular pentagon in a circle. Assume that each side of the pentagon has length 1.



- The angle at each corner of the pentagon is 108° . From this, work out the angles α , β , γ , δ , and ϵ (in that order). How long, then, is DC ?
 - Explain why $\triangle DAB$ and $\triangle ABC$ are similar. Use this fact to come up with an equation for the length of segment $y = BD$, and determine the value of y .
 - Now, using $\triangle ABF$, find $\cos(36^\circ)$, and from this, find $\sin(36^\circ)$.
- In this and the following question, you will work toward demonstrating the second Theorem of Menelaus. Using the plane diagram on the handout from class, prove the following:

$$\frac{\Gamma A}{EA} = \frac{\Gamma \Delta}{Z \Delta} \cdot \frac{BZ}{BE}$$

This is the second of the plane Menelaus theorems. (*Hint*: consider the extra line EH , parallel to ΔZ , drawn into the diagram.)

- Just as we saw in class, a plane Menelaus theorem can be extended to a spherical version. Use the spherical diagram on the same handout to demonstrate the second spherical

Menelaus Theorem:

$$\frac{\sin \Gamma A}{\sin AE} = \frac{\sin \Gamma \Delta}{\sin \Delta Z} \cdot \frac{\sin ZB}{\sin BE}$$

4. (a) We demonstrated in class that the declination of a point on the ecliptic with longitude λ may be found using the formula $\sin \delta = \sin \lambda \sin \varepsilon$, which we proved using Menelaus's Theorem. To complete the conversion to equatorial coordinates, we need to determine also the distance along the equator from Υ to the "foot" of the declination arc. This arc is known as the right ascension, α . Use one of Menelaus's Theorems to derive a formula for α .
- (b) According to my armillary sphere, on May 20 the sun's longitude will be $\lambda = 59^\circ$ (which, since that's my birthday, makes me right at the tail end of Taurus). What will be the declination and right ascension of the sun on May 20?