

# **Real Analysis**

## **Bennington College, Fall 2000**

Prerequisites:      Multivariable calculus or instructor's permission  
Meeting times:     Tuesday, Friday 8:30 AM; one additional problem-solving session?  
Instructor:         Glen Van Brummelen, [gvanbrum@bennington.edu](mailto:gvanbrum@bennington.edu), 4467 (w), 440-8142 (h)

TEXT: Introduction to Real Analysis, by Michael Schramm. Englewood Cliffs, NJ: Prentice Hall, 1996.

The techniques of differential and integral calculus are powerful scientific tools and well-established in practice, but were originally based on reasonings that are logically flawed. The increasing use of infinite series in applied mathematics led to the unsettling discoveries that apparently basic intuitions like convergence, continuity, and connectedness that underlie the calculus are subtler than they appear. The unhappy relationship between continuous and discrete magnitudes (geometry and number), an issue since ancient Greece, is the subtext for our exploration of the modern account of the number line.

Topics include the irrational numbers and the axiom of Archimedes, Dedekind cuts and the Cantor set, topological spaces, compactness, connectedness, convergence and continuity, rigorous approaches to differentiation and integration, measure theory, and Lebesgue integration.

### ***Course Work***

More or less weekly assignments will, as usual, constitute the bulk of the submitted course work. We will have regular problem-solving sessions, in which participation will be evaluated. Also, we will have the usual midterm and final synthesis assignments. These latter help to measure individual accomplishments, and hence should be done individually (without consultation). For the weekly assignments, the usual balance between fruitful interaction with classmates and submitting work you can call your own applies. If you request a letter grade, the percentage breakdown will be:

- Weekly assignments: 50%
- Midterm and final synthesis assignments: 15% and 20%
- Participation: 15%

There will be **no end-of-term application projects** in this course; the subject matter does not lend itself to them.

## *Tentative Course Outline*

- Introduction: historical background; why are formal proofs important to mathematics?
- Building Proofs (Chapter 1)
- Cardinalities and algebra (Chapters 2, 3)
- Order and neighborhoods (Chapter 4)
- Dealing with infinite sets of numbers (Chapters 5, 6, 7)
- A topological view of the real numbers (Chapter 8)
- Sequences and the Big Theorem (Chapters 9 and 10)
- Compactness and connectedness (Chapters 11 and 12)
- Differentiation and integration, revisited (Chapters 16 and 17)
- Pathologies: discontinuities and differentiability (Chapters 19 and 20)
- Integrability, measure theory, and Lebesgue integrals (Chapter 21, extra reading)

Additional topics, if time permits:

- We build the real numbers (Dedekind cuts, Chapter 22)
- The revenge of infinitesimals: nonstandard analysis (extra reading)